Extraction of effective material parameters with application to sandwich structures

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In this contribution we present a first-order numerical homogenization approach which allows for extracting effective linear elastic properties of heterogeneous materials. The approach is based on the window or self consistency method where a representative microscopic subdomain is embedded into a window of effective properties. Since these properties are not known in advance they have to be determined iteratively. For the discretization of the micro structures we use the Finite Cell Method, which is a fictitious domain method of higher-order. It is very well suited for efficiently discretizing complicated geometries stemming, for example, from tomography (CT-scans). In the numerical examples we will investigate a bending test of a sandwich plate which is composed of a polymeric core with thin faceplates made of Aluminum. Firstly, effective properties of the core are extracted and then applied to a macroscopic numerical model. The numerical results are validated by experiments.

The first-order homogenization approach based on the window method

In the scope of numerical homogenization first-order means that classical CAUCHY continua are applied on the macroscale and the microscale. Let us briefly recall the projection and homogenization rules. A formal derivation may be found in [1]. The projection rule

\[ \Delta \tilde{y} = \varepsilon M \Delta \tilde{X}, \] (1)

defines the boundary value problem for the microstructure. In (1) \( \varepsilon M \) is the macroscopic small strain tensor and \( \Delta \tilde{X} \) is the branch vector originating from the microstructure’s center. The discrete version of the homogenization reads

\[ \langle \sigma_m \rangle = \frac{1}{V_m} \sum_{i=1}^{n} r_m^{(i)} \otimes \Delta X^{(i)} \] (2)

and yields averaged stresses. Here, \( n \) is number of boundary nodes, \( V_m \) is the microstructure’s volume, and \( r_m^{(i)} \) are the nodal reaction forces. The departing point for computing the effective material parameters is a micro CT-scan resulting in the domain \( \Omega_{CT} \) from which we extract a subdomain \( \Omega_{sub} \). The subdomain is, as mentioned above, discretized by the FCM [2]. Now, one could directly apply (1) to \( \Omega_{sub} \). However, when dealing with heterogeneous materials, the local equilibrium at the boundary is not fulfilled and thus the resulting stresses respectively forces are questionable. In order to resolve this problem one embeds the subdomain into a window \( \Omega_{win} \) with thickness \( t_{win} \) and averaged stiffness, which is known as the window method [4]. In Figure 1 the procedure is sketched for the two-dimensional case, in three dimensions it works analogously.

![Fig. 1 Two-dimensional illustration of the window method](image1)

![Fig. 2 Bending test of a sandwich plate](image2)

The elastic properties of the individual cells in \( \Omega_{sub} \) correspond to the bulk value of the underlying material, and the parameters of the window are the microstructure’s effective parameters which are not known initially and therefore have to

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be determined iteratively. Within this iterative process the entries of the material matrix $C^{\text{eff}}$ are computed by numerical differentiation. To this end we choose an arbitrary macroscopic state of strain $\varepsilon$ and disturb it component-wise by $\Delta \varepsilon \neq 0$ leading to seven load cases\(^1\). Using VOIGT notation allows for column-wise storing of the load cases

$$\tilde{\varepsilon} = [ \tilde{\varepsilon}_1 \ldots \tilde{\varepsilon}_7 ] .$$

(3)

We project (3) via (1) onto the window’s boundary $\Gamma_{\text{win}}$, solve the boundary value problem, and using (2) obtain stresses

$$\langle \tilde{\sigma} \rangle = [ \langle \tilde{\sigma}_1 \rangle \ldots \langle \tilde{\sigma}_7 \rangle ]$$

(4)

for the subdomain $\Omega_{\text{sub}}$. Finally, the components of $C^{\text{eff}}$ can be computed by

$$C^{\text{eff}}_{ij} \approx \begin{cases} \frac{\langle (\tilde{\sigma}_i^1) - (\tilde{\sigma}_i^{j+1}) \rangle}{\Delta \varepsilon} & \text{if } j \leq 3 \\ \frac{1}{2} \frac{\langle (\tilde{\sigma}_i^j) - (\tilde{\sigma}_i^{j+1}) \rangle}{\Delta \varepsilon} & \text{if } j > 3 \end{cases},$$

(5)

where superscripts index the individual vector-components of the $\langle \cdot \rangle$-quantities and the factor $1/2$ is due to VOIGT notation.

**Application to bending test of a sandwich plate**  
Now, let us apply the homogenization approach to the open-cell foam core of a sandwich plate as depicted in Figure 2. The foam is made of Polyurethane and has a density of 10 pores per inch corresponding to a porosity of about 97.7%. The material parameters for the cell walls are: YOUNG’s modulus $E_c = 400 \text{ MPa}$ and POISSON’s ratio $\nu_c = 0.49$. The Aluminum faceplates have $E_f = 70 \text{ GPa}$ and $\nu_f = 0.3$. For the extraction of the core’s parameters it has turned out that a subdomain of $4 \times 4 \times 4$ foam cells embedded into a window of approximately one foam cell leads to reasonable results [5]. In Figure 3 the macroscopic mechanical model is shown, in which $H = \{40, 60, 80\} \text{ mm}$.

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**References**


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\(^1\) Note, that index $m$ and $M$ will be skipped in order to avoid overloading the notation.